Question 21

What is wrong with the following expressions? How can you correct them? (a) $C = \vec{\mathbf{A}}\vec{\mathbf{B}}$, (b) $\vec{\mathbf{C}} = \vec{\mathbf{A}}\vec{\mathbf{B}}$, (c) $C = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$, (d) $C = A\vec{\mathbf{B}}$, (e) $C + 2\vec{\mathbf{A}} = B$, (f) $\vec{\mathbf{C}} = A \times \vec{\mathbf{B}}$, (g) $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$, (h) $\vec{\mathbf{C}} = 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$, (i) $C = \vec{\mathbf{A}}/\vec{\mathbf{B}}$, and (j) $C = \vec{\mathbf{A}}/B$.

Solution

(a) Vector multiplication that results in a scalar is the dot product: $C = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

(b) Vector multiplication that results in a vector is the cross product: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

(c) The cross product yields a vector: $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

(d) Multiplying a vector by a scalar yields a vector: $\vec{\mathbf{C}} = A\vec{\mathbf{B}}$

(e) Only scalars can add with other scalars, and only vectors can add with other vectors: $C + 2A = B$ or $\vec{\mathbf{C}} + 2\vec{\mathbf{A}} = \vec{\mathbf{B}}$

(f) If A is actually a scalar, then it multiplies the vector normally. Writing it with $\times$ makes it seem like the cross product of a scalar and a vector, which is erroneous. $\vec{\mathbf{C}} = A\vec{\mathbf{B}}$ or $\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}$

(g) The dot product yields a scalar, and the cross product yields a vector. Take the magnitude of the vector to make it a scalar. $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \left|\vec{\mathbf{A}} \times \vec{\mathbf{B}}\right|$

(h) The dot product yields a scalar, and the cross product yields a vector. Take the magnitude of the vector to make it a scalar. $\left|\vec{\mathbf{C}}\right| = 2\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$

(i) There's no such thing as vector division. Change the denominator to a scalar. $\vec{\mathbf{C}} = \vec{\mathbf{A}}/B$

(j) The left side is a scalar, and the right side is a vector. Make the left side a vector as well. $\vec{\mathbf{C}} = \vec{\mathbf{A}}/B$